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ON THE USE OF MARKOVIAN APPROXIMATION IN DYNAMICS OF STOCHASTIC MEDIA

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The use of the thoery of continuous Markovian processes in problems of dynamic deformation of a stochastic elastic medium is considered. Equations are derived for the variations of geometric parameters of the wave front along a ray, which supplement the system of dynamic equations in the ray method. Certain hypotheses concerning statistical properties of a medium, which allow the formulation of the Fokker—Planck—Kolmogorov (FPK) equation for the input system, are investigated. Solutions are obtained for particular cases in the form of normal logarithmic laws of intensity distribution.

Equations which define the deformation of stochastically inhomogeneous media are statistically nonlinear, hence the closing [of equation] necessitates the introduction of hypotheses about statistical properties of considered fields [1-5]. The most complete probabilistic definition is obtained on the basis of the characteristic functional, however the methods of solving equations in variational derivatives are insufficiently developed [5-8]. Although Markovian processes make possible an effective solution of many problems involving systems with concentrated parameters [4, 9-11], they are unsuitable for dispersed systems.

The use of asymptotic (ray) methods [7, 8, 12- 16] for analyzing wave processes yields ordinary differential equations in coefficients of related expansions. Since the input system contains quantities which define variation of the front geometry of the propagating wave, it becomes necessary to introduce equations for the geometric parameters, obtained by using the variational principle of the equation for geometric parameters. The complete system of ordinary differential equations for the dynamic and geometric wave characteristics is nonlinear and depends on the form of inhomogeneity. The problem of statistical closing remains, but it becomes possible to apply the theory of Markovian processes [4, 10]. The necessary condition for the use of Markovian approximation is the existence of a small parameter which defines the relation of the scale of variation of certain functions of elastic coefficients to other characteristic dimensions of the dynamic problem. The combination of asymptotic methods with the FPK method allows a fairly complete investigation of processes of harmonic and unsteady wave propagation in certain types of stochastically inhomogeneous media.

The application of Markovian models in problems of electrodynamics and in the theory of turbulence was considered in [9, 11]. Investigation of ray diffusion in approximations of geometric optics is carried out on the assumption of smallness of ray deviation from the initial direction, hence the need to consider the complete system of geometric characteristics did not arise. In [12, 15] geometric variables were specified approximately or appeared in dynamic equations in the form of free functions.

The application of a Markovian type medium model in the calculation of the effective elastic moduli was investigated in [17].

1. Dynamics of a linear elastic stochastically inhomogeneous medium is defined by the equations

$$(\lambda + \mu)\, u_{j,ji} + \mu u_{i\ jj} + \lambda_{,i} u_{j,j} + \mu_{,j}(u_{j,i} + u_{i,j}) - \rho \ddot{u}_i = 0 \qquad (1.1)$$
$$i = 1, 2, 3$$

where u_i are components of the displacement vector, subscripts after the comma denote differentiation with respect to the corresponding coordinate x_n, $\lambda(x_n)$, and $\mu(x_n)$ are the elastic moduli stochastically dependent on space coordinates.

Equations (1.1) are stochastically nonlinear, and the closing of systems for moments necessitates the introduction supplementary assumptions. The use of method of Green's functions [4] and of the characteristic functional [6] presents considerable mathematical difficulties and the obtained approximate solutions are not always physically clear.

We seek the solution of (1.1) in the form of the ray series [7, 8, 12-16]

$$u_j(x_n, t) = \sum_{k=0}^{\infty} u_j^{(k)} f^{(k)}[t - \tau(x_n)] \quad (f^{(k)} = df^{(k-1)}/dS) \qquad (1.2)$$

where $t = \tau(x_n)$ is the position of the wave front Σ at the instant of time t, and $f^{(0)}(S)$ is a function with a singularity at $S = 0$.

For harmonic waves, always

$$u_j(x_n, t) = v_j(x_n, \gamma) \exp[i\gamma(t - \tau)]$$

and for $v_j(x_n, \gamma)$ we have the following asymptotic series in the inverse powers of frequency γ:

$$v_j(x_n, \gamma) = \sum_{k=0}^{\infty} \frac{v_j^{(k)}(x_n)}{\gamma^k}$$

Solution $u_j(x_n, t)$ may also be represented in the form (1.2), when

$$u_j^{(k)} = i^k v_j^{(k)}, \quad f^{(k)}(S) = \frac{e^{i\gamma S}}{[i\gamma]^k}, \quad i = \sqrt{-1}$$

Hence all further investigations based on solution of the form (1.2) are valid for both the unsteady waves in the wave front neighborhood and for high-frequency harmonic waves.

Using conventional methods [12, 13] we obtain the recurrent relationships between quantities $u_j^{(k)}$.

For the longitudinal ray solution we have

$$u_j^{(k)} = u_j^{(k)\perp} + \varphi^{(k)} \tau_{,j}, \quad u_j^{(k)\perp} \perp \tau_{,j}, \quad u_j^{(-2)} = u_j^{(-1)} = 0 \qquad (1.3)$$

$$(\tau_{,i})^2 = \frac{1}{c^2}, \quad \rho c^2 = \lambda + 2\mu,$$

$$u_j^{(k)\perp} = \frac{\lambda + 2\mu}{\rho(\lambda + \mu)} [M_j(u_n^{(k-1)}) - L_j(u_n^{(k-2)})]$$

$$\frac{d\varphi^{(k)}}{ds} - \varphi^{(k)}\left(\Omega + \frac{1}{2}\frac{d \ln c}{ds}\right) + \frac{c}{2\rho}[M_j(u_n^{(k)\perp}) - L_j(u_n^{(k-1)})]\,\tau_{,j} = 0$$

$$M_j(u_n) = (\lambda + \mu)[u_{i,i}\tau_{,j} + (u_i\tau_{,i})_{,j}] + \mu[u_j\tau_{,ii} + 2u_{j,i}\tau_{,i}] + \lambda_{,j}u_i\tau_{,i} + \mu_{,i}(u_i\tau_{,j} + u_j\tau_{,i})$$
$$L_j(u_n) = (\lambda + \mu)u_{i,ij} + \mu u_{j,ii} + \lambda_{,j}u_{i,i} + \mu_{,i}(u_{i,j} + u_{j,i})$$

where Ω is the mean curvature of the wave front, $\rho = \text{const}$ is the density, and d/ds denotes a derivative along the ray.

The transverse ray solution satisfies similar equations.

The main type of unstable wave discontinuity is defined by function $f^{(0)}(t - \tau)$ and the intensity variation of the longitudinal and also of the transverse discontinuity along the ray is defined by an equation of the form

$$\frac{d\omega}{ds} - \left(\Omega - \frac{1}{2}\frac{d\ln c}{ds}\right)\omega = 0, \quad \omega = |u_j^{(0)}| \tag{1.4}$$
$$c = \sqrt{\Lambda/\rho}, \quad \Lambda_1 = \lambda + 2\mu, \quad \Lambda_2 = \mu$$

Equations (1.3) and (1.4) contain as a free function the quantity Ω which is a geometric characteristic of the propagating front. Formulas for geometric parameter variation along a ray are obtained below.

We represent the equation of the wave surface in the parametric form $x_i = x_i(u^\alpha, t)$ $(\alpha = 1, 2)$ where u^α are curvilinear surface coordinates.

According to Fermat's principle a normal to the front satisfies the relationships

$$d\mathbf{v}/ds = -g^{\alpha\beta}\boldsymbol{\tau}_\beta(\ln c)_{,\alpha}, \quad d\mathbf{R}/ds = \mathbf{v} \tag{1.5}$$

where $\mathbf{v} = \{\nu_i\}$ is a normal to surface Σ, $g^{\alpha\beta} = x_\alpha{}^i x_\beta{}^i$ is the first quadratic form of Σ, $\boldsymbol{\tau}_\alpha = \{x_\alpha{}^i\}$ is the vector of the tangent to that surface along the coordinate line u^α, and $\mathbf{R} = \{x^i\}$ is the radius vector of points of the surface.

Using formulas

$$x_{\alpha\beta}{}^i = b_{\alpha\beta}\nu^i, \quad \nu_\alpha{}^i = g^{\beta\gamma}b_{\alpha\beta}x_\gamma{}^i, \quad \nu^i\nu_i = 1, \quad x_\alpha{}^i\nu_i = 0$$

of the theory of surfaces, we obtain the equations

$$db_{\alpha\beta}/ds = (\ln c)_{,\alpha\beta} + (\ln c)_{,\alpha}(\ln c)_{,\beta} - g^{\eta\delta}b_{\alpha\eta}b_{\beta\delta} \tag{1.6}$$
$$db^{\alpha\beta}/ds = g^{\alpha\eta}g^{\beta\delta}[(\ln c)_{,\eta\delta}(\ln c)_{,\eta} + (\ln c)_{,\delta}] + 3g_{\eta\delta}b^{\alpha\eta}b^{\beta\delta}$$
$$dg_{\alpha\beta}/ds = -2b_{\alpha\beta}, \quad dg^{\alpha\beta}/ds = 2b^{\alpha\beta}$$
$$d\boldsymbol{\tau}_\alpha/ds = (\ln c)_{,\alpha}\mathbf{v} - g^{\delta\gamma}b_{\delta\alpha}\boldsymbol{\tau}_\gamma \tag{1.7}$$

which define the variation of covariant and contravariant components of the first and second quadratic forms of Σ and of the quantity τ_α along a ray.

Formulas (1.5)-(1.7) with suitable initial conditions completely determine the evolution of the ray and frontgeometry in the propagation process.

We introduce the invariants $2\Omega = b_{\alpha\beta}g^{\alpha\beta}$, $2K = 4\Omega^2 - b_{\alpha\beta}b^{\alpha\beta}$ (the mean and Gaussian curvatures, respectively) and, taking into account equalities (1.6), obtain

$$d\Omega/ds = 2\Omega^2 - K + c_{,\alpha\beta}g^{\alpha\beta}/2c \tag{1.8}$$
$$dK/ds = 2\Omega K + 2\Omega c_{,\alpha\beta}g^{\alpha\beta}/c - c_{,\alpha\beta}b^{\alpha\beta}/c$$

A comma in the subscript in formulas (1.6)-(1.8) denotes covariant differentiation.

Formulas (1.4)—(1.8) constitute a closed system of first order differential equations, which for specified initial conditions and known velocity c defines the kinematics and dynamics in the zero approximation. To obtain solutions of higher orders it is necessary to use Eq. (1.3).

2. We represent the input equations (1.3)—(1.8) in the form

$$d\,(\xi_i^{(\alpha\beta)})/ds = \Phi_i\,(\xi_k^{(\alpha\beta)},\ \eta_k^{(\alpha\beta)}),\quad i = 1, 2, \ldots, n \tag{2.1}$$

where $\eta_k^{(\alpha\beta)}$ is a random function of known probability characteristics. In this case $\ln c$, $d\,(\ln c)/ds$, $(\ln c)_{,\alpha}$, and $(\ln c)_{,\alpha\beta}$ are taken for such functions.

Further transformations of (2.1) are linked with the introduction of formulas in which the random functions $\eta_k^{(\alpha\beta)}$ are expressed in terms of subsidiary functions $q_i^{(\alpha\beta)}$ with the following properties: 1) $q_i^{(\alpha\beta)}(s)$ form a random Gaussian field, 2) $\langle q_i^{(\alpha\beta)}\rangle = 0$, and 3) $\langle q_i^{(\alpha\beta)}(s)\, q_j^{(\gamma\delta)}(s')\rangle = \delta\,(s - s')\, A_{ij}^{(\alpha\beta\gamma\delta)}$, where $A_{ij}^{(\alpha\beta\gamma\delta)}$ is a quantity which defines the power of white noise $q_i^{(\alpha\beta)}$ at points of surface Σ. Along the ray that power is assumed constant.

If $\eta_j^{(\alpha\beta)}(s)$ are steady random functions of s with rational-fractional spectral densities, the equations for η_j assume the form

$$d\,(\eta_j^{(\alpha\beta)})/ds = F_j\,(\eta_k^{(\alpha\beta)}) + G_j\,(\eta_k^{(\alpha\beta)})\, q_k^{(\alpha\beta)},\quad j = 1, 2, \ldots, l \tag{2.2}$$

The use of subsidiary functions with zero correlation time for defining the mathematical model of stochastically inhomogeneous medium in the form (2.2) yields equations in $\xi_m^{(\alpha\beta)}$ which belong to the class of systems that describe continuous multivariate Markovian processes [4, 10].

System (2.1), (2.2), after renumbering, assumes the form

$$d\,(\xi_m^{(\alpha\beta)})/ds = W_m\,(\xi_k^{(\alpha\beta)}) + f_m\,(q_k^{(\alpha\beta)},\ \xi_k^{(\alpha\beta)}),\quad m = 1, 2, \ldots, l + n \tag{2.3}$$

Let us consider the case when $F_j \equiv 0$ and $G_j = 1$

$$d\,(\eta_j^{(\alpha\beta)})/ds = q_j^{(\alpha\beta)}(s) \tag{2.4}$$

It follows from (2.4) that perturbations $\eta_j^{(\alpha\beta)}$ are represented by the conventional Wiener process [18] with constant mathematical expectation $\langle\eta_j^{(\alpha\beta)}\rangle = \eta_{j0}^{(\alpha\beta)}$ ($\eta_{j0}^{(\alpha\beta)}$ is the value of $\eta_j^{(\alpha\beta)}$ when $s = 0$). For Markovian processes $\eta_j^{(\alpha\beta)}(s)$ we obtain $\langle q_i^{(\alpha\beta)}(s)\,\eta_j^{(\gamma\delta)}(s')\rangle = 0$ when $s' \leqslant s$, and the longitudinal correlation which in consequence of (2.4) satisfies the equation $d\langle\eta_i^{(\alpha\beta)}(s)\,\eta_j^{(\gamma\delta)}(s')\rangle / ds = 0$, is of the form

$$\langle\eta_i^{(\alpha\beta)}(s)\,\eta_j^{(\gamma\delta)}(s')\rangle = \langle\eta_i^{(\alpha\beta)}(s')\,\eta_j^{(\gamma\delta)}(s')\rangle = \eta_{i0}^{(\alpha\beta)}\eta_{j0}^{(\gamma\delta)} + A_{ij}^{(\alpha\beta\gamma\delta)}s'$$

The use of more complex models reduces to the problem of obtaining from a white noise a random functions with specific probability characteristics which are solved by means of the forming filter (2.2) [4, 18].

The complete system (2.3) which determines parameter variation of the internal geometry of the front and ray trajectories for the Wiener model

$$\frac{d(\ln c)_{,\alpha}}{ds} = q_\alpha(s), \quad \frac{d(\ln c)_{,\alpha\beta}}{ds} = q_{\alpha\beta}(s)$$

is of the form

$$\begin{aligned}
&\xi_i^{(\alpha\beta)} = \{(\ln c)_{,\alpha},\ (\ln c)_{,\alpha\beta},\ b_{\alpha\beta},\ b^{\alpha\beta},\ g_{\alpha\beta},\ g^{\alpha\beta},\ \mathbf{R},\ \mathbf{v},\ \boldsymbol{\tau}_\alpha\} \\
&W_i^{(\alpha\beta)} = \{0,\ 0,\ W',\ W'',\ -2\, b_{\alpha\beta},\ 2b^{\alpha\beta},\ \mathbf{v},\ - \\
&\quad g^{\alpha\beta}\boldsymbol{\tau}_\beta (\ln c)_{,\alpha},\ (\ln c)_{,\alpha}\mathbf{v} - g^{\beta\gamma} b_{\delta\alpha}\boldsymbol{\tau}_\gamma\} \\
&f_i^{(\alpha\beta)} = \{q_\alpha,\ q_{\alpha\beta},\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0\} \\
&W' = (\ln c)_{,\alpha\beta} + (\ln c)_{,\alpha} (\ln c)_{,\beta} - g^{\eta\delta} b_{\alpha\eta} b_{\beta\delta} \\
&W'' = g^{\alpha\eta} g^{\beta\gamma} [(\ln c)_{,\eta\gamma} + (\ln c)_{,\eta} (\ln c)_{,\gamma}] + 3\, g_{\eta\delta} b^{\alpha\eta} b^{\beta\delta}
\end{aligned} \tag{2.5}$$

and for a Markovian model of the type (2.4)

$$d(\ln c)/ds = q(s), \quad d(N_{\alpha\beta})/ds = q_{\alpha\beta}(s)$$
$$N_{\alpha\beta} = c_{,\alpha\beta}/c = (\ln c)_{,\alpha\beta} + (\ln c)_{,\alpha} (\ln c)_{,\beta}$$

the equations for variation of intensity reduce to the form (2.3) in which $\xi_i^{(\alpha\beta)}$, $W_i^{(\alpha\beta)}$, and $f_i^{(\alpha\beta)}$ are defined as follows:

$$\begin{aligned}
&\xi_i^{(\alpha\beta)} = \{N_{\alpha\beta},\ \chi,\ b_{\alpha\beta},\ g_{\alpha\beta}\}, \quad \chi = \ln \omega \\
&W_i^{(\alpha\beta)} = \{0,\ {}^1\!/_2\, g^{\alpha\beta} b_{\alpha\beta},\ N_{\alpha\beta} - g^{\eta\delta} b_{\alpha\eta} b_{\beta\delta},\ -2\, b_{\alpha\beta}\} \\
&f_i^{(\alpha\beta)} = \{q_{\alpha\beta},\ {}^1\!/_2\, q,\ 0,\ 0\}
\end{aligned} \tag{2.6}$$

The reduction of dynamic equations (1.3) - (1.8) to the scheme of Markovian processes is based on the representation of input equations in the form (2.3), (2.6) and (2.5). This and the assumptions about the δ- correlation properties of certain functions of elastic coefficients make it possible to use the theory of continuous Markovian processes [4, 11] according to which the combined density of distribution of the indicated characteristics satisfies the FPK equation.

Assuming that the intensity is an element of the multivariate Markovian process (2.3), (2.6), for the distribution density $P(\chi, N_{\alpha\beta}, b_{\alpha\beta}, g_{\alpha\beta}, s)$ we obtain

$$\begin{aligned}
&\frac{\partial P}{\partial s} + \Omega \frac{\partial P}{\partial \chi} - 4\Omega P + (N_{\alpha\beta} - g^{\eta\delta} b_{\alpha\eta} b_{\beta\delta}) \frac{\partial P}{\partial b_{\alpha\beta}} - 2 b_{\alpha\beta} \frac{\partial P}{\partial g_{\alpha\beta}} - \\
&\pi \left[D_{11} \frac{\partial^2 P}{\partial \chi^2} + (D_{12}^{(\alpha\beta)} + D_{21}^{(\alpha\beta)}) \frac{\partial^2 P}{\partial \chi\, \partial N_{\alpha\beta}} + D_{22}^{(\alpha\beta\gamma\delta)} \frac{\partial^2 P}{\partial N_{\alpha\beta}\, \partial N_{\gamma\delta}} \right] = 0
\end{aligned} \tag{2.7}$$

where D_{11}, $D_{12}^{(\alpha\beta)}$, $D_{21}^{(\alpha\beta)}$, $D_{22}^{(\alpha\beta\gamma\delta)}$ are constant spectral densities of perturbations $d(\ln c)/ds$ and $d(N_{\alpha\beta})/ds$.

It should be noted that for non-Gaussian fluctuations of functions $q^{(\alpha\beta)}(s)$ it is theoretically possible to obtain equations of the FPK type for the related characteristic functional, but the introduction of the latter considerably complicates solution of the problem.

Below we adduce the solutions of (2.7) in some particular cases.

3. Let us consider the problem of determining the probability density for the wave intensity in the case when the front curvatures are determinate functions of s. This condition imposes restrictions on the properties of a medium which is assumed to be random inhomogeneous along s and homogeneous along the front.

The solution of system (1.8) assumes the form

$$\Omega(s) = (\Omega_0 - K_0 s)\,\varkappa(s), \quad K(s) = K_0 \varkappa(s) \qquad (3.1)$$
$$\varkappa(s) = (1 - 2\,\Omega_0 s + K_0 s^2)^{-1}$$

where Ω_0 and K_0 are the mean and the Gaussian curvatures of the front at the initial instant $s = s_0$.

Let us consider two kinds of media: 1) $d(\ln c)/ds = q(s)$ and 2) $d(\ln c)/ds = \ln c + q(s)$, where $q(s)$ represents white noise of constant power $4N$. Then for the Wiener model 1) the logarithm of intensity satifies the stochastic equation

$$d\chi(s)/ds = (\Omega_0 - K_0 s)\,\varkappa(s) - 1/2\, q(s) \qquad (3.2)$$

The FPK equation for the density distribution of probability $P(\chi, s)$ which corresponds to (3.2) is of the form

$$\frac{\partial P(\chi, s)}{\partial s} + (\Omega_0 - K_0 s)\,\varkappa(s)\,\frac{\partial P(\chi, s)}{\partial \chi} - \frac{N}{2}\,\frac{\partial^2 P(\chi, s)}{\partial \chi^2} = 0 \qquad (3.3)$$

Solving (3.3) with the initial condition $P(\chi, 0) = \delta(\chi - \chi_0)$, where χ_0 is the logarithm of initial density, we obtain

$$P(\chi, s) = \sqrt{\frac{1}{2\pi N s}}\,\exp\left\{-\frac{[\chi - \chi_0 - {}^1/_2 \ln \varkappa(s)]^2}{2Ns}\right\}$$

Thus for χ we obtain the normal distribution law with mathematical expectation $\chi_0 + 1/2 \ln \varkappa(s)$ and dispersion Ns.

To determine longitudinal correlation χ we multiply Eq. (3.2) by $\chi(s')$ $(s' < s)$ and average

$$d\langle \chi(s)\,\chi(s')\rangle/ds = (\Omega_0 - K_0 s)\,\varkappa(s)\,\langle \chi(s')\rangle + 1/2\,\langle q(s)\,\chi(s')\rangle \qquad (3.4)$$

Taking into account that $\langle q(s)\,\chi(s')\rangle = 0$, we obtain for (3.4) a solution of the form

$$\langle \chi(s)\,\chi(s')\rangle = D(s') - 1/2\,[\chi_0 + 1/2 \ln \varkappa(s')]\,[\ln \varkappa(s') - \ln \varkappa(s)] \qquad (3.5)$$
$$D(s') = \langle \chi(s')\,\chi(s')\rangle = 1/2\,Ns' + [\chi_0 + 1/2 \ln \varkappa(s')]^2$$

The univariate density of probability distribution of intensity ω may be represented in the form

$$P^{(1)}_{(\omega, s)} = \frac{\omega_0}{\sqrt{2\pi N s}\,\omega}\,\exp\left\{-\frac{[\ln(\omega/\omega_0) - {}^1/_2 \ln \varkappa(s)]^2}{2Ns}\right\} \qquad (3.6)$$

The expression for $P^{(2)}(\omega, s)$ when the perturbation $\ln c$ satisfies condition 2)

$$P^{(2)}_{(\omega, s)} = \frac{\omega_0}{\sqrt{2\pi}\,\omega\,\sigma(s)}\,\exp\left\{-\frac{[\ln(\omega/\omega_0) - {}^1/_2 \ln \varkappa(s)]^2}{2\sigma^2(s)}\right\} \qquad (3.7)$$
$$\sigma^2(s) = {}^1/_2\,[1 - \exp(-2Ns)]$$

is similarly derived.

Formulas (3.6) and (3.7) show that for both models of medium the wave intensity obey the logarithmic-normal distribution law. Note that the process $\omega^{(1)}$ does not have a steady mode. In the case of a plane wave $(\Omega_0 = 0,\ K_0 = 0)$ distribution

$P^{(2)}(\omega, s)$ becomes steady when $s \to \infty$.

Curves (*1* and *2*) of variation $P^{(1)}(\omega)$ and $P^{(2)}(\omega)$ are shown in Fig. 1 for a plane wave and $sN = 0.125$. Values $\omega = 0.882\omega_0$ ($\omega = 0.895\,\omega_0$) correspond to the maximum density of distribution $P^{(1)}$ ($P^{(2)}$). The dash line represents the steady state distribution $P^{(2)}(\omega)$. In the other limit case ($s = 0$) $P^{(1)} = P^{(2)} = \delta(\omega - \omega_0)$. Variation of density $P^{(1)}(\omega)$ for $sN = 0.25$ and $sN = 1$ is represented by curves *3* and *4*, respectively.

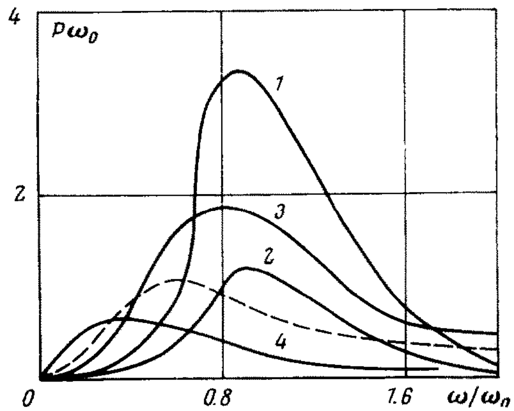

Fig. 1

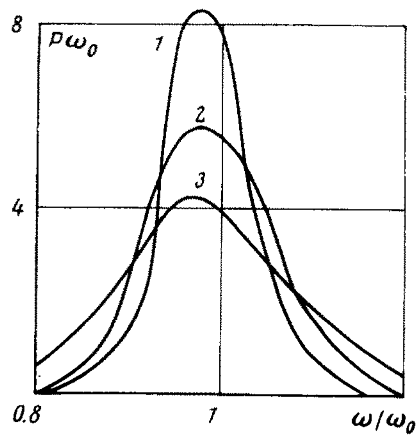

Fig. 2

The effect of power N on the pattern of intensity distribution was investigated in the case of a spherical wave of initial radius R_0 with $s / R_0 = 0.01$. The curves calculated for models 1) and 2) are the same to within 0.01. They are shown in Fig. 2 by curves 1, 2, and 3 for $R_0 N$ equal 0.25, 0.5 and 1, respectively.

Investigation of probabilistic properties of higher order solutions requires the use of recurrent formulas (1.3) which constitute a system of ordinary differential equations of the type (2.3) and reduce to a scheme of the Markovian process similarly to the system of Eqs. (2.3), (2.6) and (2.7) for the determination of intensity. Each step of the recurrent process is accompanied by an increase by unity of the order of the derivative along the ray of quantities that define the model of medium with the δ- correlation function $q(s)$. Hence the k-th order solution requires the introduction of new, as compared to the $(k-1)$-st order, assumptions concerning the statistical properties of certain functions of the medium parameters. Equations for subsequent approximations for probability densities are generally of the operator form and contain variational derivatives.

Finally, we turn to equations of rays, which can be presented in the form $d\nu_i / ds = (\ln c)_{,i} - \nu_i \nu_j (\ln c)_{,j}$, $dx_i / ds = \nu_i$ [9, 11], and are investigated separately from the surface geometry. Passing to ray coordinates u^α, s related to the front geometry makes it possible to present these relationships in the form of Eqs. (1.5) which have to be considered together with (1.6) and (1.7). In the Wiener model with fixed u^α the ray equations are reduced to the set (2.3)—(2.5). The assumption that surface Σ in the neighborhood of point $u^\alpha = \text{const}$ is a plane yields the equations $d\mathbf{R}_\perp / ds = \boldsymbol{\nu}(s)$, $d\boldsymbol{\nu} / ds = d(\ln c) / d\mathbf{R}_\perp$ ($\mathbf{R}_\perp$ is the plane transverse dislocation) that are simpler than (1.5)—(1.7) and coincide with formulas in [9, 11] derived in the case of small angular deflections of rays and are suitable for the complete analysis in the

Markovian approximation. In the general case formulas (1.5)-(1.7) are closed and can be considered independently from Eqs. (1.3) and (1.4).

The above analysis shows that it is possible to obtain on the basis of the Fermat principle equations for the geometric characteristics of the front. These equations close the system of related dynamic formulas. On fairly general assumptions it is possible to use the FPK method for defining the nonlinear input system. The distribution of elastic parameters is then close to the logarithmic-normal, which conforms to experimental data on rock. The obtained front intensity distributions in the considered examples are also closely observed in [6].

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